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Degree of Polarization in Anisotropic Single-Mode Optical Fibers: Theory

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Abstract—**The degree of polarization for propagation waves in anisotropic single-mode fibers is formulated in terms of light source spectrum, incident polarization condition, and fiber parameters. The polarization degree deterioration is based on the incident wave split into two eigenpolarization modes inherent in the fiber. Since the two eigenpolarization modes have different group velocities from each other, the degree of polarization is degraded when both of the modes are excited. Polarization degree is preserved when only one of the eigenpolarization modes is excited. The degradation is determined by the mutual correlation function γ, between the two modes, which depends on the light source spectra, fiber polarization dispersion, and fiber length.**

I. Introduction

PRESERVATION of the optical polarized state in fibers is essential to realize coherent optical transmission using the frequency or phase shift keying and heterodyne detection scheme [1]. Fibers which preserve linear polarization have conventionally been studied by many researchers [2], [3]. Recently, a proposal has been made to transmit a circularly polarized light in a twisted single-mode fiber [4]. These works seem to be based on an idea that incident polarized light should be transmitted without polarization conversion.

The state of incident polarization may be changed by a scattering process, external mechanical stresses, ambient changes, and other causes. The apparent degradation in the degree of polarization can be recovered by adjusting retardation at the fiber output [5]. Only intrinsic degradation remains after ideal phase compensation is carried out. It has been shown that proper incident polarization states exist which preserve a high degree of polarization [6], [7]. Polarization mode dispersion has been discussed to explain the polarization degree degradation for linear polarization incidence [8].

The purpose of this paper is to formulate the inherent polarization degree in anisotropic single-mode optical fibers. The degradation mechanism is based on an assumption that any incident polarization state is split into two orthogonal eigenpolarization modes [7], which propagate at different group velocity values. The degree of polarization depends on a mutual correlation function between two eigenpolarization modes.

Section II explains properties of the eigenpolarization modes which play an important role in preserving the degree of polarization. The mathematical definition of the eigenpolarizations, their physical features, their vector expressions, and expansion using them are presented. Section III provides the degree of polarization in terms of light source spectrum and fiber parameters by using a coherency matrix. The degree of polarization is compared with several source spectral profiles. In Section IV, the eigenpolarization modes and the polarization degree are described for a twisted elliptical core fiber, as an example of anisotropic single-mode fibers.

II. Eigenpolarization Modes in Anisotropic Single-Mode Fibers

A. Definition of Eigenpolarization Modes

Polarization evolution in anisotropic single-mode fibers has been treated by means of the modified coupled-mode equations containing coupling coefficients N_{ij} [9]. Eigenpolarization modes correspond to eigenstates with particular shapes and propagation constants, independent of propagation length z, as have been theoretically investigated [10]. A mathematical outline of the eigenpolarization modes will be briefly described.

Electric fields in anisotropic single-mode fibers can be repre-

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sented as $E = A(z)e_1 + B(z)e_2$ with two orthonormalized eigenfunctions e_1 and e_2. When coupling coefficients N_{ij} are independent of z in a particular coordinate system, the field amplitudes $A(z)$ and $B(z)$ can be expressed by $A(z) = A_i \exp(-j\beta_i z)$ and $B(z) = B_i \exp(-j\beta_i z)$, where

$$\beta_i = (\tfrac{1}{2})\,[(N_{11} + N_{22}) \pm \{(N_{11} - N_{22})^2 + |2N_{12}|^2\}^{1/2}] \quad (i = 1, 2) \quad (1)$$

and

$$\frac{B_i}{A_i} = \frac{2N_{12}}{(N_{11} - N_{22}) \pm \{(N_{11} - N_{22})^2 + |2N_{12}|^2\}^{1/2}} \quad (i = 1, 2). \quad (2)$$

The index $i = 1$ ($i = 2$) corresponds to the upper (lower) sign in (1) and (2). The β_i and B_i/A_i indicate mathematical eigenvalues and eigenfunctions, respectively. It has been shown that the eigenstate has a physical significance incorporated with polarization behavior [7]. These eigenstates will be called eigenpolarization modes hereafter. Birefringence $\delta\beta$ or propagation constant difference between the two eigenpolarization modes is defined by

$$\delta\beta = \beta_1 - \beta_2 = [(N_{11} - N_{22})^2 + |2N_{12}|^2]^{1/2}. \quad (3)$$

B. Complex Representation of Eigenpolarization Modes

Assume that coupling coefficients N_{11} and N_{22} are real, and N_{12} is a complex value defined by $N_{12} = N_r + jN_i$. The states of two eigenpolarizations are represented as

$$\zeta_1 \equiv \xi_1 + j\eta_1 = E_{1y}/E_{1x} = (\xi_1^2 + \eta_1^2)^{1/2} \exp(j\delta_1)$$

and

$$\zeta_2 \equiv \xi_2 + j\eta_2 = E_{2y}/E_{2x} = (\xi_2^2 + \eta_2^2)^{1/2} \exp[j(\delta_1 + \pi)] \quad (4)$$

with

$$\tan\delta_1 = \eta_1/\xi_1 = \eta_2/\xi_2 = N_i/N_r.$$

Here, the E_{ix} and E_{iy} are electric components in Cartesian coordinate.

Fig. 1 illustrates the shape of two eigenpolarization modes schematically. It can be found that two major axis angles ψ_i of the polarization ellipses satisfy

$$\psi_2 = \psi_1 + m(\pi/2) \quad (m\text{: integer}) \quad (5)$$

with the help of the standard text [11] and (4). This implies that the principal axes of two eigenpolarization ellipses are parallel or perpendicular to each other. Ellipticity of the polarization is defined by the ratio of the semiaxes as

$$\tan\chi_i = b_i/a_i \quad (i = 1, 2). \quad (6)$$

For two eigenpolarization ellipticities one obtains

$$\tan\chi_2 = -1/\tan\chi_1. \quad (7)$$

The minus sign indicates that two eigenpolarizations rotate in opposite directions. The inverse relation between two polarization ellipticities means that the major axes of the two polarizations are perpendicular to each other.

In summary, two eigenpolarization modes, which belong to different eigenvalues, have the following features. 1) Ellipticities of two eigenpolarizations are identical with each other. 2) The major axes of the polarization ellipses are mutually perpendicular. 3) The endpoints of their electric vectors rotate oppositely. 4) Ellipticity of the eigenpolarization modes depends on perturbations applied to the single-mode fiber [10].

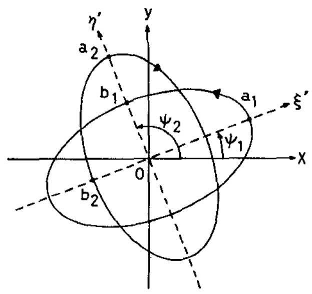


Fig. 1. Schematics of two eigenpolarization modes. a_i and b_i denote the semimajor and semiminor axis lengths, respectively. ψ_i indicates the orientation of major axis for each polarization ellipse with respect to the fixed x axis.

C. Vector Representation for Eigenpolarization Modes

Let the unit vectors along the fixed x and y axes be e_x and e_y, respectively. A pair of vectors for eigenpolarization modes are represented by

$$e_1 = Xe_x + Ye_y$$
$$e_2 = -Y^*e_x + X^*e_y \quad (8)$$

where

$$Y = X(\xi_1^2 + \eta_1^2)^{1/2} \exp(j\delta_1)$$
$$|X| = 1/[1 + (\xi_1^2 + \eta_1^2)]^{1/2}. \quad (9)$$

Here, each vector e_i is normalized so as to satisfy

$$|e_i|^2 = |X|^2 + |Y|^2 = 1 \quad (10)$$

and fulfills an orthogonal relation: $e_1 \cdot e_2^* = 0$ and $e_1^* \cdot e_2 = 0$. The asterisk indicates complex conjugation.

D. Expansion of Elliptically Polarized Light in Terms of Eigenpolarization Modes

An arbitrary incident elliptically polarized light E is expanded into a pertinent combination of two eigenpolarization mode vectors e_1 and e_2. This can be shown as

$$E = E_1e_1 + E_2e_2,$$
$$= (E_1X - E_2Y^*)e_x + (E_1Y + E_2X^*)e_y \quad (11)$$

where E_1 and E_2 stand for the expansion coefficients. Incident polarization components can be expressed by

$$E_y/E_x = (\tan\alpha)\exp(j\delta). \quad (12)$$

Here, $\tan\alpha$ is the incident field component ratio and δ is the phase difference. Comparison of (12) with (11) leads to

$$|E_1|^2 = \frac{|X|^2 + |Y|^2\tan^2\alpha + 2\,\mathrm{Re}\,[XY^*(\tan\alpha)\exp(j\delta)]}{1 + \tan^2\alpha}$$

$$|E_2|^2 = \frac{|Y|^2 + |X|^2\tan^2\alpha - 2\,\mathrm{Re}\,[XY^*(\tan\alpha)\exp(j\delta)]}{1 + \tan^2\alpha} \quad (13)$$

where Re indicates that the real part of the square bracket is to be taken. The incident light power is normalized to satisfy $|E|^2 = 1$.

Polarization ellipticity χ and major axis angle ψ for the polarization ellipse are suitable parameters for relating to experimental data. Parameters α and δ presented in (12) are related with χ and ψ as

$$\alpha = \arctan\left[\frac{|1/\cos 2\psi| - |\cos 2\chi|}{|1/\cos 2\psi| + |\cos 2\chi|}\right]^{1/2} \tag{14}$$

$$\delta = \arccos\left[\frac{|\cos 2\chi/\cos 2\psi|^2 - |\cos 2\chi|^2}{|1/\cos 2\psi|^2 - |\cos 2\chi|^2}\right] \tag{15}$$

by solving usual results [11] reversely. For linear polarization incidence, namely $\chi = 0$, α is reduced to linear polarization angle ψ with respect to the x axis.

III. Degree of Polarization

The mechanism explaining degradation in the degree of polarization will be outlined first. An arbitrary incident elliptical polarization is expanded into a proper combination of two eigenpolarization modes, as shown in (11). The two eigenpolarization modes propagate at different group velocities from each other. Even when the ideal phase compensation is carried out at the fiber output, the two eigenpolarizations cannot be made a linear polarization at the same time, giving rise to a spurious orthogonal polarization component.

The degree of polarization P is defined by the ratio of polarized component intensity I_{pol} to total intensity I_{tot}. The degree of polarization can be expressed, with the aid of coherency matrix $\hat{J}$, as [11]

$$P = I_{pol}/I_{tot} = [1 - 4(\det \hat{J})/(\operatorname{tr} \hat{J})^2]^{1/2}. \tag{16}$$

Here, det and tr denote the determinant and trace, respectively. The degree of polarization is independent of choice of coordinate system, since det $\hat{J}$ and tr $\hat{J}$ remain unchanged with respect to a transformation in coordinates. Coherency matrix $\hat{J}$ is defined by

$$\hat{J} = \langle E \cdot E^\dagger \rangle = \begin{pmatrix} J_{\xi\xi}, J_{\xi\eta} \\ J_{\eta\xi}, J_{\eta\eta} \end{pmatrix} \tag{17}$$

where ξ and η represent the components for an arbitrary coordinate system. The dagger indicates Hermitian transpose and the angle bracket implies time average. When a field is stationary and ergodic, the ensemble average is replaced by the time average. The field components of E can be represented in terms of a complex analytic signal [11]. Analytic signal $V(t)$ is expressed as

$$V(t) = 2\int_0^\infty v(\omega) \exp(j\omega t)\, d\omega. \tag{18}$$

Amplitude spectrum $v(\omega)$ is represented, by means of Fourier transform, as

$$v(\omega) = \begin{cases} \displaystyle\int_{-\infty}^{\infty} V(t) \exp(-j\omega t)\, dt; & \omega \geqq 0 \\ 0; & \omega < 0. \end{cases} \tag{19}$$

The incident light is assumed to be quasi-monochromatic. Column vector E_0 of incident light can be represented by

$$E_0 = \begin{pmatrix} V_{0\xi}(t) \\ V_{0\eta}(t) \end{pmatrix} \tag{20}$$

with

$$V_{0\xi}(t) = \frac{1}{(1 + \tan^2\alpha)^{1/2}}\, 2\int_0^\infty v(\omega) \exp(j\omega t)\, d\omega$$

$$V_{0\eta}(t) = \frac{(\tan\alpha)\exp(j\delta)}{(1 + \tan^2\alpha)^{1/2}}\, 2\int_0^\infty v(\omega) \exp(j\omega t)\, d\omega. \tag{21}$$

Here, $\tan\alpha$ stands for the field amplitude ratio of η to ξ components and δ denotes the phase difference. The factor $1/(1 + \tan^2\alpha)^{1/2}$ is introduced to normalize the incident light power. Input light power I_0 can be evaluated as

$$I_0 = \operatorname{tr}\langle E_0 \cdot E_0^\dagger\rangle = S_0 \tag{22}$$

with

$$S_0 \equiv 2\int_0^\infty |v(\omega)|^2\, d\omega. \tag{23}$$

In deriving (23), (18), (19), and (21) were employed.

Any incident elliptically polarized light E_0 is split into two eigenpolarization mode vectors, e_1 and e_2, as shown in (11). The envelope of the fields is essential to explain the polarization degree deterioration. The vector components for two eigenpolarization modes can be written as

$$e_1 = \begin{pmatrix} e_{1\xi}(t) \\ e_{1\eta}(t) \end{pmatrix} \quad e_2 = \begin{pmatrix} e_{2\xi}(t) \\ e_{2\eta}(t) \end{pmatrix} \tag{24}$$

where

$$e_{1\xi}(t) = X \cdot 2\int_0^\infty v(\omega) \cdot \exp[j(\omega - \omega_0)\{t - (d\beta_1/d\omega)z\}]\, d\omega$$

$$e_{1\eta}(t) = Y \cdot 2\int_0^\infty v(\omega) \cdot \exp[j(\omega - \omega_0)\{t - (d\beta_1/d\omega)z\}]\, d\omega$$

$$e_{2\xi}(t) = -Y^* \cdot 2\int_0^\infty v(\omega) \cdot \exp[j(\omega - \omega_0)\{t - (d\beta_2/d\omega)z\}]\, d\omega$$

$$e_{2\eta}(t) = X^* \cdot 2\int_0^\infty v(\omega) \cdot \exp[j(\omega - \omega_0)\{t - (d\beta_2/d\omega)z\}]\, d\omega. \tag{25}$$

Here, X and Y are the field components of the mode 1, as shown in (9). The $\beta_i(\omega)$ indicates propagation constant for each mode. It is assumed that the spectral spread of the light source is sufficiently smaller than central angular frequency ω_0. Assuming that total optical power is preserved, even after splitting into two eigenpolarization modes, one obtains

$$|E_1|^2 + |E_2|^2 = 1. \tag{26}$$

Here, the power immediately after incidence on the fiber was evaluated by using eigenpolarization mode components in (25) with $z = 0$ and employing a normalization relation in (10).

The coherency matrix elements at the fiber output can be represented by

$$J_{\xi\xi} = \langle |E_1 e_{1\xi} + E_2 e_{2\xi}|^2 \rangle$$
$$J_{\eta\eta} = \langle |E_1 e_{1\eta} + E_2 e_{2\eta}|^2 \rangle$$
$$J_{\xi\eta} = J_{\eta\xi}^* = \langle (E_1 e_{1\xi} + E_2 e_{2\xi})(E_1 e_{1\eta} + E_2 e_{2\eta})^* \rangle. \quad (27)$$

Fiber output optical power I_f can be presented by

$$I_f = \mathrm{tr}\, \hat{J} = S_0 \quad (28)$$

by making use of (10) and (26). The optical power is also conserved at the fiber output. The $\det \hat{J}$ at the output is obtained as

$$\det \hat{J} = (S_0 - |S_1|^2) |E_1|^2 |E_2|^2 \quad (29)$$

with

$$S_1 \equiv 2 \int_0^\infty |v(\omega)|^2 \exp [j(\omega - \omega_0)\delta\tau_g z] \, d\omega. \quad (30)$$

Here, $\delta\tau_g$ stands for the polarization dispersion or the group delay difference between two eigenpolarization modes and is defined by

$$\delta\tau_g = \frac{d\beta_1}{d\omega} - \frac{d\beta_2}{d\omega} = \frac{1}{c} \cdot \frac{d}{dk} (\delta\beta). \quad (31)$$

In the above equation, c is the light velocity in free space, k is the vacuum wave number, and $\delta\beta$ is the birefringence defined by (3). Parameter S_1 depends on the light source spectral distribution, fiber polarization dispersion, and fiber length.

The degree of polarization for anisotropic single-mode fibers is obtained, by substitution of (28) and (29) into (16), as

$$P = [1 - (1 - |\gamma|^2)(4|E_1|^2|E_2|^2)]^{1/2} \quad (32)$$

with

$$\gamma \equiv S_1/S_0. \quad (33)$$

In (32), $(4|E_1|^2|E_2|^2)$ and $|\gamma|$ depend on the incident condition and light source coherency, respectively. When only one eigenpolarization mode is excited at the fiber input, the degree of polarization $P = 1$ is always maintained during fiber propagation under idealized conditions. This is due to the fact that autocorrelation functions for each eigenpolarization mode satisfy

$$|\gamma_{\xi\eta}^{(i)}| = \left| \frac{\langle e_{i\xi} e_{i\eta}^* \rangle}{\sqrt{\langle |e_{i\xi}|^2 \rangle \langle |e_{i\eta}|^2 \rangle}} \right| = 1 \quad (i = 1, 2). \quad (34)$$

Coherency is perfectly kept between intramodal components of one eigenpolarization mode.

On the other hand, the degree of polarization P takes the minimum value $|\gamma|$ for equally split powers: $|E_1|^2 = |E_2|^2 = 0.5$. For mutual correlation functions between two eigenpolarization modes

$$\left| \frac{\langle e_{i\{\xi \atop \eta\}} e_{j\{\xi \atop \eta\}}^* \rangle}{\sqrt{\langle |e_{i\{\xi \atop \eta\}}|^2 \rangle \langle |e_{j\{\xi \atop \eta\}}|^2 \rangle}} \right| = |\gamma| \quad (i \neq j) \quad (35)$$

holds. Here, { } implies either of the components is to be chosen. When absolute value of the mutual correlation function approaches unity, the degree of polarization also tends toward unity.

Mutual correlation function γ is connected with the light source spectral intensity $|v(\omega)|^2$ by Fourier transform, as can be found from (30). The γ value is given by

$$\gamma = \sin(\delta\omega \cdot \delta\tau_g \cdot z)/(\delta\omega \cdot \delta\tau_g \cdot z) \quad (36)$$

for a rectangular source spectrum with spectral width $2\delta\omega$ and central angular frequency ω_0. This functional form is the same presented by Rashleigh *et al.* [8]. Let $2 \cdot \delta\omega$ be the full width at half the maximum intensity, hereafter. The condition $\omega_0 \gg \delta\omega$ is assumed throughout this paper. For a Gaussian spectrum defined by $|v(\omega)|^2 = \exp[-(\ln 2)\{(\omega - \omega_0)/\delta\omega\}^2]$, whose profile has been observed in a gas laser [12], we have

$$\gamma = \exp\left[-\left(\frac{\delta\omega \cdot \delta\tau_g \cdot z}{2\sqrt{\ln 2}}\right)^2\right]. \quad (37)$$

For a Lorentzian spectrum defined by $|v(\omega)|^2 = \delta\omega^2/[(\omega - \omega_0)^2 + \delta\omega^2]$, whose line shape has been demonstrated in a semiconductor laser [13],

$$\gamma = \exp[-(\delta\omega \cdot \delta\tau_g \cdot z)] \quad (38)$$

is obtained. These three kinds of γ are real values depending on the product $\delta\omega \cdot \delta\tau_g \cdot z$ alone.

Fig. 2 compares with the mutual correlation function values for the above three shapes with a single spectral peak. In the neighborhood of $x \equiv \delta\omega \cdot \delta\tau_g \cdot z = 0$, a high coherency is kept for rectangular and Gaussian spectra, but a rapid degradation in coherency appears for a Lorentzian shape. Values of x giving $\gamma = 0.9$ are 0.79, 0.54, and 0.11, respectively, for rectangular, Gaussian, and Lorentzian shapes. For example, $x = 0.001$ is obtained for $\delta\omega = 1$ GHz, $\delta\tau_g = 1$ ps/km, and $z = 1$ km.

Consider a light source which has two spectral intensity peaks with central frequency spacing ω_s and peak intensity ratio I_s. When a spectral distribution is represented by addition of Gaussian functions: $|v(\omega)|^2 = \exp[-(\ln 2)\{(\omega - \omega_0)/\delta\omega_1\}^2] + I_s \exp[-(\ln 2)\{(\omega - \omega_0 - \omega_s)/\delta\omega_2\}^2]$ with the spectral half widths, $\delta\omega_1$ and $\delta\omega_2$, the mutual correlation function is given by

$$\gamma = \frac{\delta\omega_1 \exp\left[-\left(\frac{\delta\omega_1 \cdot \delta\tau_g \cdot z}{2\sqrt{\ln 2}}\right)^2\right] + I_s \cdot \delta\omega_2 \exp\left[-\left(\frac{\delta\omega_2 \cdot \delta\tau_g \cdot z}{2\sqrt{\ln 2}}\right)^2\right] \exp(j\omega_s \cdot \delta\tau_g \cdot z)}{\delta\omega_1 + I_s \cdot \delta\omega_2}. \quad (39)$$

When Lorentzian shapes are prescribed as $|v(\omega)|^2 = \delta\omega_1^2/$

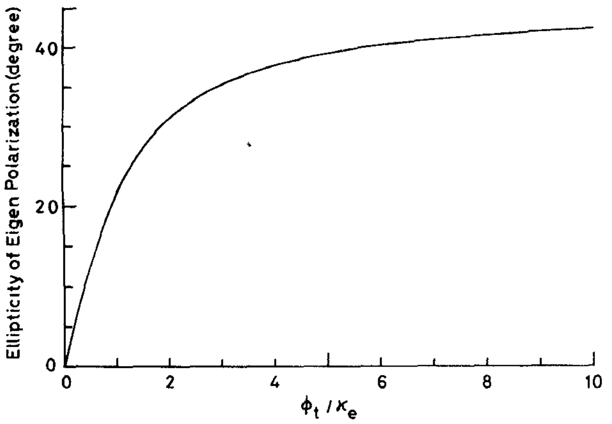


Fig. 4. Polarization ellipticity of eigenpolarization modes in twisted elliptical core fibers. κ_e and ϕ_t denote the core ellipticity effect and fiber twist rate, respectively. $\alpha \equiv GC/n = 0.074$.

Vector components for the eigenpolarization modes, corresponding to (9), can be described as

$$X = [\kappa_e + \{\kappa_e^2 + (\phi_t - \kappa_t)^2\}^{1/2}]/N_p$$
$$Y = j(\phi_t - \kappa_t)/N_p$$
$$N_p \equiv \sqrt{2}\,[\kappa_e^2 + (\phi_t - \kappa_t)^2 + \kappa_e\{\kappa_e^2 + (\phi_t - \kappa_t)^2\}^{1/2}]^{1/2}. \tag{43}$$

The optical powers for ξ and η components are given by $P_\xi = |X|^2$ and $P_\eta = |Y|^2$. For a sufficiently small twist rate, these powers are approximated by

$$P_\xi \cong 1 - (\tfrac{1}{4})(\phi_t/\kappa_e)^2\{1 - (GC/n)\}^2$$
$$P_\eta \cong (\tfrac{1}{4})(\phi_t/\kappa_e)^2\{1 - (GC/n)\}^2 \qquad (\phi_t/\kappa_e \ll 1) \tag{44}$$

to the second order in (ϕ_t/κ_e). On the other hand, when the twist rate ϕ_t is much larger than κ_e,

$$P_\xi \cong (\tfrac{1}{2})\,[1 + \{1 + (GC/n)\}/(\phi_t/\kappa_e)]$$
$$P_\eta \cong (\tfrac{1}{2})\,[1 - \{1 + (GC/n)\}/(\phi_t/\kappa_e)] \qquad (\phi_t/\kappa_e \gg 1) \tag{45}$$

hold to the second order in (κ_e/ϕ_t).

Fig. 6 illustrates optical powers for ξ and η components as functions of the ratio ϕ_t/κ_e. $P_\xi \cong 0.946$ is obtained even when $\phi_t/\kappa_e = 0.5$, since eigenpolarization modes are approximated with linear polarizations for $\phi_t/\kappa_e \ll 1$. When fiber twist rate ϕ_t by far exceeds κ_e, optical powers of eigenpolarization modes are distributed almost equally in the ξ and η axes. Approximate values given in (44) show a relative error of about 1 percent at $\phi_t/\kappa_e = 0.5$, while those in (45) produce an about 4 percent error at $\phi_t/\kappa_e = 2$.

Polarization dispersion is obtained as [9], [15]

$$\delta\tau_g = \delta\tau_g^{(e)}\,[\kappa_e/\{\kappa_e^2 + (\phi_t - \kappa_t)^2\}^{1/2}]$$
$$\delta\tau_g^{(e)} = \frac{\kappa_e}{ck} \cdot \frac{(d/dv)\,[vG(v)]}{G(v)}. \tag{46}$$

Polarization dispersion $\delta\tau_g^{(e)}$ indicates a parameter caused by the core ellipticity effect alone. Polarization dispersion $\delta\tau_g$ tends towards zero with increasing twist rate ϕ_t because the birefringence due to fiber twist hardly depends on the wavelength.

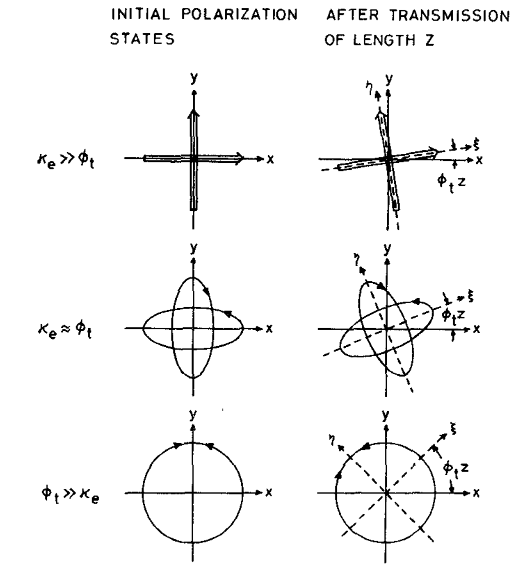


Fig. 5. Polarization evolution of eigenpolarization modes classified by the magnitude of core ellipticity effect κ_e and fiber twist rate ϕ_t.

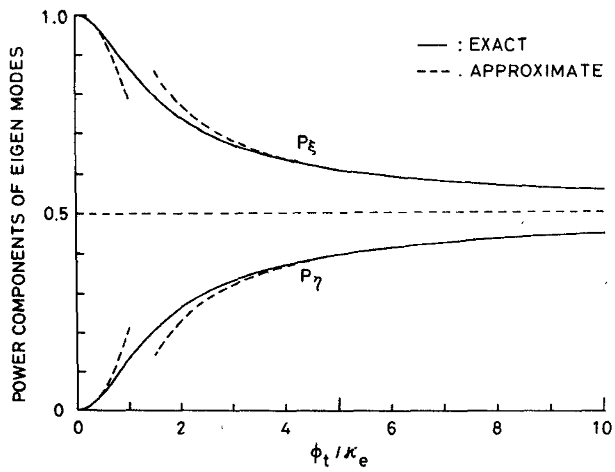


Fig. 6. Optical power distributed along rotation coordinates, ξ and η, for eigenpolarization mode. Broken curves indicate approximate values given in (44) and (45). $\alpha \equiv GC/n = 0.074$.

When elliptically polarized light, represented by $E_\eta/E_\xi = (\tan\alpha)\exp(j\delta)$, is incident on the fiber, expansion coefficients for two eigenpolarization modes are given by

$$|E_1|^2 = \frac{X^2 + |Y|^2\tan^2\alpha + 2X|Y|\tan\alpha\cdot\sin\delta}{1 + \tan^2\alpha}$$
$$|E_2|^2 = \frac{|Y|^2 + X^2\tan^2\alpha - 2X|Y|\tan\alpha\cdot\sin\delta}{1 + \tan^2\alpha}. \tag{47}$$

The component power ratio depends on the incident polarized state and ratio (ϕ_t/κ_e).

The degree of polarization can be obtained by inserting (43), (46), and (47) into (32). Fig. 7(a) and (b) show the degree of polarization P for linearly and circularly polarized light excitations, respectively. This figure illustrates the value of P for three kinds of light source spectral functions, presented in (36)-(38). For the linear polarization excitation, P takes the minimum value near a particular ratio ϕ_t/κ_e and is raised to $P = 1$ with leaving its minimum value. The ratio ϕ_t/κ_e giving

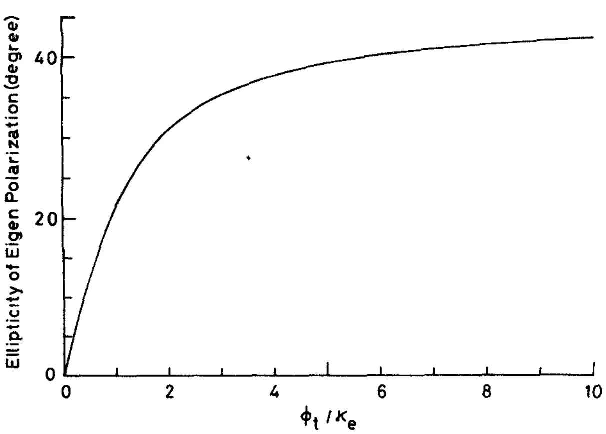


Fig. 4. Polarization ellipticity of eigenpolarization modes in twisted elliptical core fibers. κ_e and ϕ_t denote the core ellipticity effect and fiber twist rate, respectively. $\alpha \equiv GC/n = 0.074$.

Vector components for the eigenpolarization modes, corresponding to (9), can be described as

$$X = [\kappa_e + \{\kappa_e^2 + (\phi_t - \kappa_t)^2\}^{1/2}]/N_p$$
$$Y = j(\phi_t - \kappa_t)/N_p$$
$$N_p \equiv \sqrt{2}\,[\kappa_e^2 + (\phi_t - \kappa_t)^2 + \kappa_e\{\kappa_e^2 + (\phi_t - \kappa_t)^2\}^{1/2}]^{1/2}. \tag{43}$$

The optical powers for ξ and η components are given by $P_\xi = |X|^2$ and $P_\eta = |Y|^2$. For a sufficiently small twist rate, these powers are approximated by

$$P_\xi \cong 1 - (\tfrac{1}{4})(\phi_t/\kappa_e)^2\{1 - (GC/n)\}^2$$
$$P_\eta \cong (\tfrac{1}{4})(\phi_t/\kappa_e)^2\{1 - (GC/n)\}^2 \qquad (\phi_t/\kappa_e \ll 1) \tag{44}$$

to the second order in (ϕ_t/κ_e). On the other hand, when the twist rate ϕ_t is much larger than κ_e,

$$P_\xi \cong (\tfrac{1}{2})\,[1 + \{1 + (GC/n)\}/(\phi_t/\kappa_e)]$$
$$P_\eta \cong (\tfrac{1}{2})\,[1 - \{1 + (GC/n)\}/(\phi_t/\kappa_e)] \qquad (\phi_t/\kappa_e \gg 1) \tag{45}$$

hold to the second order in (κ_e/ϕ_t).

Fig. 6 illustrates optical powers for ξ and η components as functions of the ratio ϕ_t/κ_e. $P_\xi \cong 0.946$ is obtained even when $\phi_t/\kappa_e = 0.5$, since eigenpolarization modes are approximated with linear polarizations for $\phi_t/\kappa_e \ll 1$. When fiber twist rate ϕ_t by far exceeds κ_e, optical powers of eigenpolarization modes are distributed almost equally in the ξ and η axes. Approximate values given in (44) show a relative error of about 1 percent at $\phi_t/\kappa_e = 0.5$, while those in (45) produce an about 4 percent error at $\phi_t/\kappa_e = 2$.

Polarization dispersion is obtained as [9], [15]

$$\delta\tau_g = \delta\tau_g^{(e)}\,[\kappa_e/\{\kappa_e^2 + (\phi_t - \kappa_t)^2\}^{1/2}]$$
$$\delta\tau_g^{(e)} = \frac{\kappa_e}{ck} \cdot \frac{(d/dv)\,[vG(v)]}{G(v)}. \tag{46}$$

Polarization dispersion $\delta\tau_g^{(e)}$ indicates a parameter caused by the core ellipticity effect alone. Polarization dispersion $\delta\tau_g$ tends towards zero with increasing twist rate ϕ_t because the birefringence due to fiber twist hardly depends on the wavelength.

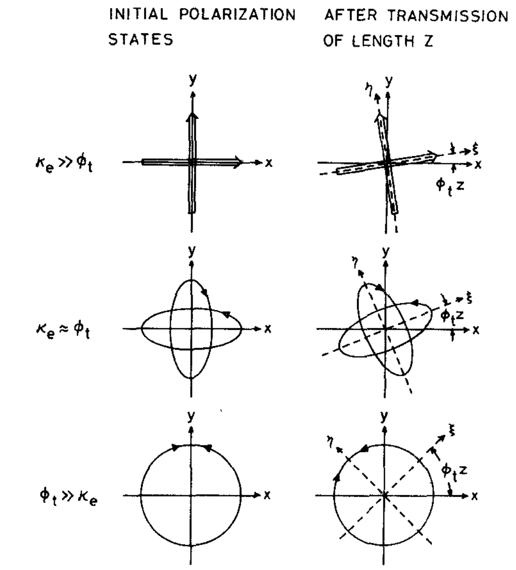


Fig. 5. Polarization evolution of eigenpolarization modes classified by the magnitude of core ellipticity effect κ_e and fiber twist rate ϕ_t.

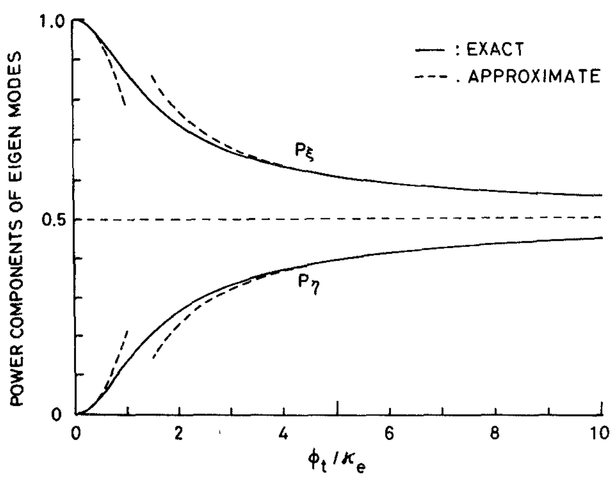


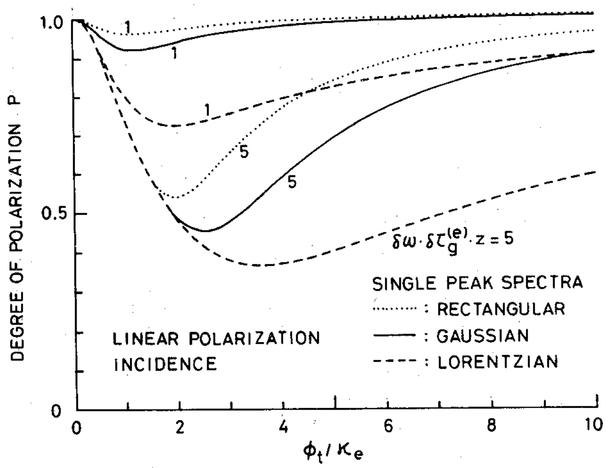
Fig. 6. Optical power distributed along rotation coordinates, ξ and η, for eigenpolarization mode. Broken curves indicate approximate values given in (44) and (45). $\alpha \equiv GC/n = 0.074$.

When elliptically polarized light, represented by $E_\eta/E_\xi = (\tan\alpha)\exp(j\delta)$, is incident on the fiber, expansion coefficients for two eigenpolarization modes are given by

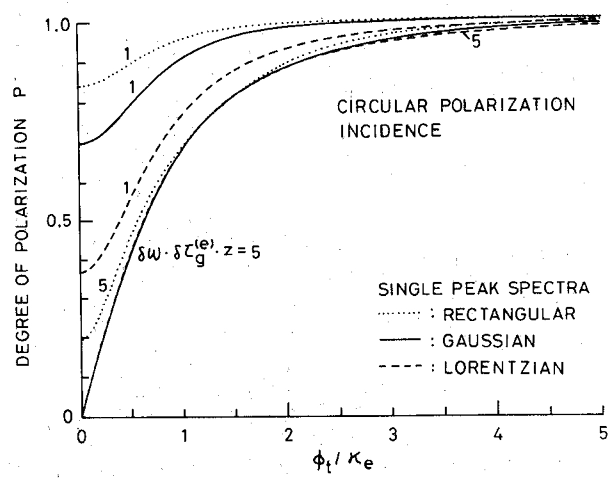
$$|E_1|^2 = \frac{X^2 + |Y|^2\tan^2\alpha + 2X|Y|\tan\alpha \cdot \sin\delta}{1 + \tan^2\alpha}$$
$$|E_2|^2 = \frac{|Y|^2 + X^2\tan^2\alpha - 2X|Y|\tan\alpha \cdot \sin\delta}{1 + \tan^2\alpha}. \tag{47}$$

The component power ratio depends on the incident polarized state and ratio (ϕ_t/κ_e).

The degree of polarization can be obtained by inserting (43), (46), and (47) into (32). Fig. 7(a) and (b) show the degree of polarization P for linearly and circularly polarized light excitations, respectively. This figure illustrates the value of P for three kinds of light source spectral functions, presented in (36)-(38). For the linear polarization excitation, P takes the minimum value near a particular ratio ϕ_t/κ_e and is raised to $P = 1$ with leaving its minimum value. The ratio ϕ_t/κ_e giving



(a)



(b)

Fig. 7. Degree of polarization for linear and circular polarization incidences as a function of the ratio ϕ_t/κ_e. Spectral functions are presented in (36)–(38). $\alpha \equiv GC/n = 0.074$. (a) Linearly polarized light incidence. (b) Circularly polarized light incidence.

the minimum P increases with a parameter $\delta\omega \cdot \delta\tau_g^{(e)} \cdot z$. If the spectral halfwidth $\delta\omega$ is identical for the three spectral shapes, the rectangular shape shows a larger P value than the Gaussian and Lorentzian shapes.

When circular polarization is launched, as shown in Fig. 7(b), the P value increases monotonically with ratio ϕ_t/κ_e. The tendency of the P value for the three spectral shapes is similar to that in Fig. 7(a). When the fiber twist effect predominates over the core ellipticity effect, the P value tends toward unity for any incident polarized state.

The achievement of $P = 1$ appears near $\phi_t/\kappa_e = 0$ for the linear polarization incidence and at a sufficiently large ϕ_t/κ_e for the circular polarization, as shown in Fig. 7(a) and (b). This phenomena can be explained by the fact that the eigenpolarization modes are linear polarizations for an untwisted fiber and are circular polarizations for a strongly twisted fiber. The convergence to $P = 1$ at large ϕ_t/κ_e reflects that polarization dispersion $\delta\tau_g$, between the two eigenpolarization modes, approaches zero with increasing ϕ_t/κ_e. The lower P value for Lorentzian spectral shape results from the fact that the Lorentzian shape shows a large tail in the optical frequency domain.

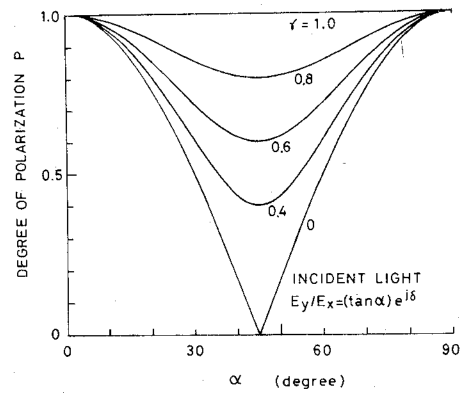


Fig. 8. Degree of polarization in an elliptical core fiber as functions of mutual correlation function γ and incident polarized state.

In particular, the degree of polarization in an elliptically deformed core fiber can be represented as

$$P = [1 - (1 - |\gamma|^2) \sin^2 (2\alpha)]^{1/2} \tag{48}$$

for an incident condition given in (12). Equation (48) implies that the P depends on incident electric component ratio $\tan\alpha$ but not on phase difference δ. Fig. 8 shows the degree of polarization as functions of the mutual correlation function γ and incident polarized state. The P value takes the minimum value, which equals $|\gamma|$, at $\alpha = 45°$.

For linear polarization incidence, α agrees with linear polarization angle ψ with respect to the x axis. When incident light is linearly polarized along the x or y axis, namely $\psi = 0°$ or $90°$, degree of polarization $P = 1$ is always preserved, in spite of incident light spectral shape, polarization dispersion, and fiber length. This fact is the basis of usual single-polarization maintaining fibers, which have asymmetrical index profiles and are uniaxially stressed. When a linear polarization, with its orientation $\psi_i = 45°$, is incident on a fiber and light source spectral shape is rectangular, (48) reduces to the previous result [8].

Equation (48) holds not only for an elliptical core fiber but also for a general linear retardation fiber, if polarization dispersion is appropriately chosen for each fiber.

V. Conclusion

The degree of polarization in anisotropic single-mode optical fibers has been formulated in terms of light source spectrum and fiber parameters with the aid of a coherency matrix. The deterioration mechanism in the polarization degree is based on the assumption that an arbitrary incident polarized light is split into two separate eigenpolarization modes, which propagate at different group velocity values.

When only one of the eigenpolarization modes is launched at the fiber input, degree of polarization $P = 1$ is always achieved. On the other hand, when equal power is excited in two eigenpolarization modes, the degree of polarization is minimized. The minimum value depends on mutual correlation function $\gamma = S_1/S_0$ between the two eigenpolarization modes. The γ depends on light source spectral shape, fiber polarization dispersion $\delta\tau_g$ and fiber length z. Usually, the degree of polarization P is kept high for rectangular and Gaussian spectral functions, whereas P is rapidly reduced for a

Lorentzian shape, provided that the spectral halfwidths are identical.

The eigenpolarization modes and the degree of polarization for a twisted elliptical core fiber have been studied as an example of anisotropic fibers. Ellipticity and principal axis orientation for the eigenpolarizations are determined by the fiber core ellipticity and twist rate. When a sufficiently large twist is applied, the degree of polarization amounts to unity for any incident polarized state. This is due to the polarization dispersion, between the two eigenpolarization modes, tending toward zero with increasing fiber twist.

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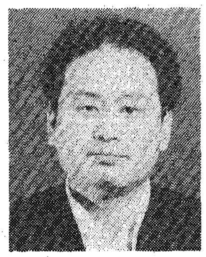

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Tatsuya Kimura (S'63–M'68–SM'78), for a photograph and biography, see p. 65 of the January 1982 issue of *IEEE J. Quantum Electron.*